Mały ilustrowany słownik terminów matematycznych

MAŁGORZATA MIKOŁAJCZYK

63. Trigonometic identities – Tożsamości trygonometryczne

THE QUADRANT RULE

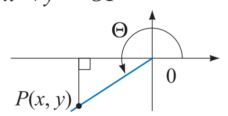
$s+$ $c-$ $t-$	$s+$ $c+$ $t+$
$s-$ $c-$ $t+$	$s-$ $c+$ $t-$

or

s	s c t
t	c

THE „SQUARED RATIO" GROUP/THE PYTHAGOREAN GROUP

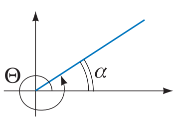
$x^2 + y^2 = OP^2$



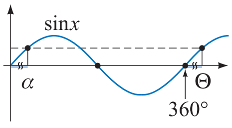
$$\cos^2\Theta + \sin^2\Theta \equiv 1$$
$$1 + \tan^2\Theta \equiv \sec^2\Theta$$
$$\cot^2\Theta + 1 \equiv \mathrm{cosec}^2\Theta$$

THE ASSOCIATED ACUTE ANGLE RULE

$\alpha = \Theta - 360°$

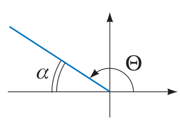


$\sin\Theta = \sin(\Theta - 360°)$
$\cos\Theta = \cos(\Theta - 360°)$
$\tan\Theta = \tan(\Theta - 360°)$

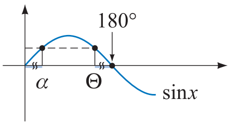


1st Quadrant

$\alpha = 180° - \Theta$

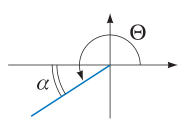


$\sin\Theta = \sin(180° - \Theta)$
$\cos\Theta = -\cos(180° - \Theta)$
$\tan\Theta = -\tan(180° - \Theta)$

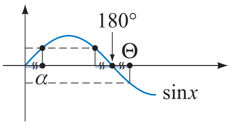


2nd Quadrant

$\alpha = \Theta - 180°$

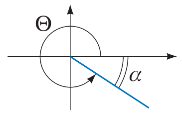


$\sin\Theta = -\sin(\Theta - 180°)$
$\cos\Theta = -\cos(\Theta - 180°)$
$\tan\Theta = \tan(\Theta - 180°)$

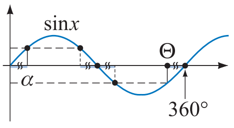


3rd Quadrant

$\alpha = 360° - \Theta$

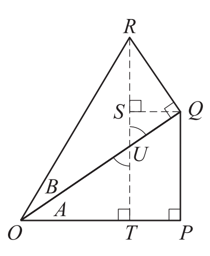


$\sin\Theta = -\sin(360° - \Theta)$
$\cos\Theta = \cos(360° - \Theta)$
$\tan\Theta = -\tan(360° - \Theta)$



4th Quadrant

THE COMPOUND ANGLE IDENTITIES



$$\sin(A+B) = \frac{TR}{OR} = \frac{TS+SR}{OR} = \frac{PQ+SR}{OR} = \frac{PQ}{OQ}\cdot\frac{OQ}{OR} + \frac{SR}{QR}\cdot\frac{QR}{OR} = \sin A\cos B + \cos A\sin B$$

$$\sin(A \pm B) \equiv \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) \equiv \cos A\cos B \mp \sin A\sin B$$
$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

THE DOUBLE ANGLE FORMULAE

$\sin 2A = 2\sin A\cos A \qquad \tan 2A \equiv \dfrac{2\tan A}{1-\tan^2 A}$

$\cos 2A = \cos^2 A - \sin^2 A$

THE HALF ANGLE FORMULAE

$\sin A \equiv \dfrac{2t}{1+t^2} \qquad \tan A \equiv \dfrac{2t}{1-t^2}$

$\cos A \equiv \dfrac{1-t^2}{1+t^2} \qquad \text{for} \quad t = \tan\dfrac{A}{2}$

THE FACTOR FORMULAE

$2\sin A\cos B \equiv \sin(A+B)+\sin(A-B)$

$2\cos A\sin B \equiv \sin(A+B)-\sin(A-B)$

$2\cos A\cos B \equiv \cos(A+B)+\cos(A-B)$

$-2\sin A\sin B \equiv \cos(A+B)-\cos(A-B)$

$\sin P+\sin Q \equiv 2\sin\dfrac{P+Q}{2}\cos\dfrac{P-Q}{2}$

$\sin P-\sin Q \equiv 2\cos\dfrac{P+Q}{2}\sin\dfrac{P-Q}{2}$

$\cos P+\cos Q \equiv 2\cos\dfrac{P+Q}{2}\cos\dfrac{P-Q}{2}$

$\cos P-\cos Q \equiv -2\sin\dfrac{P+Q}{2}\sin\dfrac{P-Q}{2}$

SMALL ANGLES FORMULAE

$\sin\Theta \approx \Theta$ for $-6° < \Theta < 6°$

$\tan\Theta \approx \Theta$ correct to 3 s.f.

$\cos\Theta \approx 1 - \dfrac{\Theta^2}{2}$

READ ME

$2\sin A\cos B \equiv \sin(A+B)+\sin(A-B)$ – twice sine cosine equals sine of sum plus sine of difference

$\sin P+\sin Q \equiv 2\sin\dfrac{P+Q}{2}\cos\dfrac{P-Q}{2}$ – sum of sines equals twice sine of semi-sum cosine of semi-difference

WORD PUZZLES

Label true or false

□ In the first quadrant all six trigonometric ratios are positive.

□ The relationship $\tan x \cdot \cos x = \sin x$ is a trigonometric identity i.e. it holds for any angle.

□ The trigonometric identities are very useful in the solution of certain trig equations.

Choose the proper word

□ Not every trigonometric EQUATION/EQUALITY is an IDENTITY/IDENTICALITY, as sometimes only DISTINCT/CHOSEN values of a VARIABLE/UNKNOWN satisfy them.

□ The COPOUND/COMPOSITE angle formulae when used for two equal angles give the DOUBLE/HALF angle identities.

□ The FACTOR/PRODUCT formulae is the set of identities which converts expressions such as $\sin A + \sin B$ into a PRODUCT/FACTOR, so it FACTORIZES/PRODUCES these expressions.

Fill in the gaps (one word per gap).

□ In the .... for the sine of the .... of angles A and B it is enough to replace the angle of B by $(-B)$ to show the identity for the .... of angles A and B.

□ Furthermore, after replacing A by $(\frac{\pi}{2} - A)$ one is left with the identity $\cos(A+B) \equiv$ ....

□ The ..... angle formulae are the most useful of all .... identities, being a powerful for .... trig functions.

Select the phrase that best completes the sentence: *must be*, *cannot be*, *can be but needn't be.*

□ In the first quadrant the angle is acute and, since that, their numerical values .... obtained directly from tables..

□ As $\sin(A+B)$ does not equal $\sin A + \sin B$, the sine function .... distributive versus addition.

□ The .... formulae are derived from the .... angle group of identities by adding or .... them side by side.

Tell if the sentence is true ALWAYS/SOMETIMES/NEVER.

□ A particular value of one trigonometric ratio applies to an infinite set of angles.

□ $2\cos\alpha$ equals $\sin(\alpha + 30°)$.

□ It is better to memorise trig identities in words rather than as symbols.

TIME FOR PROBLEMS

1) Solve the equation $2\cos 2x - \sin x = 1$ for values of x between 0 and 2π.

2) Prove that $(1-\cos A)(1+\sec A) \equiv \sin A\tan A$.

3) If $\sin\alpha = \dfrac{1}{3}$ and α is obtuse, find $\cos\alpha$ and $\cot\alpha$ without using tables or calculator.

4) Evaluate:
a) $\sin 75°$, b) $\cos 105°$, c) $\tan(-15°)$,
d) $\cos 80°\cos 20° + \sin 80°\sin 20°$.

COMING SOON

SOLVING TRIANGLES – Rozwiązywanie trójkątów

MAŁGORZATA MIKOŁAJCZYK

pracownik Instytutu Matematycznego
Uniwersytetu Wrocławskiego